Math for Machine Learning – Quiz

August 2025

1. Consider the matrix
$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}.$$

   (a) Compute the rank of A.

   (b) Does the system $A\mathbf{x} = \mathbf{b}$ have (i) no solution, (ii) a unique solution, or (iii) infinitely many solutions?

2. Consider the function
$$f(x) = x^3 - 3x.$$

   (a) Compute the derivative $f'(x)$.

   (b) Find all critical points of $f(x)$.

   (c) Determine which critical points are local minima and which is a local maximum.

3. In December 2022, Austin was experiencing a surge of COVID cases during the Omicron wave. Suppose:

   - The prevalence of COVID in Austin at that time was about 15%, i.e. $P(\text{COVID}) = 0.15$.
   - A rapid antigen test has:
     - Sensitivity = 90%, i.e. $P(\text{Positive} \mid \text{COVID}) = 0.90$,
     - Specificity = 95%, i.e. $P(\text{Negative} \mid \text{No COVID}) = 0.95$.

   You take the test and the result is **positive**.

   Question: Using Bayes' theorem, compute the probability that you actually had COVID given the positive test result. Show your steps.