M408C: Problem Set 9

Problem 1

Find the local maximum and minimum values of

$$f(x) = \frac{x^2}{x-1}$$

using both the first and the second derivative tests.

Problem 2

Use the guidelines from class to sketch the graph of these functions:

$$f(x) = x^3 + 3x^2, \quad f(x) = \frac{x}{x^2-4}, \quad f(x) = \frac{x-3}{\sqrt{x}}.$$

Problem 3

Use the guidelines from class to sketch the graph of the these functions:

$$f(x) = \sin^3(x), \quad f(x) = \frac{e^x}{x^2}.$$

Problem 4

Verify that the following functions satisfy the hypothesis of the mean value theorem on the given interval.

$$f(x) = 2x^2 - 3x + 1 \text{ on } [0,2], \quad f(x) = \ln(x) \text{ on } [1,4].$$

Then find the number c that satisfy the conclusion of the mean value theorem.

Problem 5

Find the following limits:

$$\lim_{x\to 3} \frac{x-3}{x^2-9}, \quad \lim_{x\to 0} \frac{x-\sin(x)}{x-\tan(x)}, \quad \lim_{x\to 0} \frac{x\sin(x-1)}{2x^2-x-1}, \quad \lim_{x\to \frac{\pi}{2}+} \frac{\cos(x)}{1-\sin(x)}.$$

Use l'Hopital's Rule where appropriate (if there is a more elemental method it is probably faster). If it does not apply, explain why.