M408C: Problem Set 3

Problem 1

Evaluate the following limits if they exist:

$$\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2} \right), \quad \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}.$$

Problem 2

Use the definition of continuity and the properties of limits to show that the following functions are continuous at the given number a:

$$(x + 2x^3)^4 \ a = -1, \quad \frac{t^2 + 5t}{2t + 1} \ a = 2.$$

Problem 3

For the function f whose graph is illustrated below, calculate

$$\lim_{x \to -\infty} f(x), \quad \lim_{x \to \infty} f(x), \quad \lim_{x \to 1} f(x), \quad \lim_{x \to 3} f(x).$$

Problem 4

Find the limits or show that they do not exist:

$$\lim_{x \to \infty} \frac{3x - 2}{2x + 1}, \quad \lim_{x \to \infty} \left(\sqrt{2x^2 + x} - 3x \right), \quad \lim_{x \to \infty} \frac{1 - x^2}{x^3 - x + 1}.$$

Problem 5

Find the derivative of the following functions using the definition:

$$3x - 4, \quad x^2 - 2^3, \quad \frac{1}{\sqrt{t}}.$$

State the domain of the function and the domain of its derivative.