

## M408C: Problem Set 13

### Problem 1

Evaluate the following integrals:

$$\int x\sqrt{1-x^2} dx, \quad \int x^2 e^{x^3} dx, \quad \int_0^3 \frac{1}{5x+1} dx.$$

### Problem 2

Sketch the region enclosed by the given curves and find its area:

- $y = e^x$ ,  $y = x^2 - 1$ ,  $x = -1$ ,  $x = 1$ .
- $y = \sin(x)$ ,  $y = x$ ,  $x = \frac{\pi}{2}$ ,  $x = \pi$ .
- $y = (x - 2)^2$ ,  $y = x$ .

### Problem 3

Sketch the region enclosed by the given curves and find its area:

- $y = 12 - x^2$ ,  $y = x^2 - 6$ .
- $y = \cos(x)$ ,  $y = 2 - \cos(x)$ ,  $x = 0$ ,  $x = \frac{\pi}{2}$ .
- $y = \sqrt{x-1}$ ,  $x - y = 1$ .

### Problem 4

Sketch the solid obtained by rotating the region bounded by the given curves about the  $x$ -axis, then calculate its volume:

- $y = x + 1$ ,  $y = 0$ ,  $x = 0$ ,  $x = 2$ .
- $y = e^x$ ,  $y = 0$ ,  $x = -1$ ,  $x = 1$ .

## Problem 5

Sketch the solid obtained by rotating the region bounded by the given curves about the  $y$ -axis, then calculate its volume:

- $y = \sqrt[3]{x}$ ,  $y = 0$ ,  $x = 1$ .
- $y = x^3$ ,  $y = 0$ ,  $x = 1$ ,  $x = 2$ .

## Problem 6

Find the average value of the function on the given interval:

- $f(x) = 3x^2 + 8x$  on  $[-1, 2]$ .
- $f(x) = e^{\sin(x)} \cos(x)$  on  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .