

M408C: Problem Set 12

Problem 1

Evaluate the integrals using the fundamental theorem of calculus:

$$\int_1^3 \frac{x^3 - 2x^2 - x}{x^2} dx, \quad \int_0^4 2^x dx, \quad \int_{1/2}^{\frac{1}{\sqrt{2}}} \frac{4}{\sqrt{1-x^2}} dx.$$

Problem 2

Find the most general indefinite integrals:

$$\int \sqrt[4]{x^5} dx, \quad \int \frac{1}{1+x^2} dx, \quad \int \frac{\sin(2x)}{\sin(x)} dx.$$

Problem 3

A particle is moving along a straight line with the given data. Find the position $x(t)$ of the particle expressed as a function of t when

1. $v(t) = 2t - \frac{1}{1+t^2}$, $x(0) = 1$.
2. $a(t) = \sin(t) + 3 \cos(t)$, $x(0) = 0$ and $v(0) = 2$.

Here $v(t)$ denotes the velocity at time t and $a(t)$ the acceleration at time t .

Problem 4

Evaluate the integrals by making the given substitutions:

$$\int \cos(2x) dx \quad u = 2x, \quad \int x e^{-x^2} dx \quad u = -x^2, \quad \int \sin^2(\theta) \cos(\theta) d\theta \quad u = \sin(\theta).$$

Problem 5

Find the area of the following shaded regions:

