M408C: Problem Set 10

Problem 1

Find the followin limits:

$$\lim_{x \to 0} \frac{e^{2t} - 1}{\sin(t)}, \quad \lim_{x \to \infty} \frac{\ln(x)}{\sqrt{x}}, \quad \lim_{x \to \infty} x^{e^{-x}}.$$

Use l'Hopital's Rule where appropriate (if there is a more elemental method it is probably faster). If it does not apply, explain why.

Problem 2

Find two numbers whose difference is 100, and the product is minimized.

Problem 3

Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

Problem 4

Find the most general antiderivative of the following functions

$$f(x) = 4x + 7, \quad f(x) = (x - 5)^2, \quad f(x) = \frac{1}{5} - \frac{2}{x}.$$

Problem 5

Estimate the area under the graph of $f(x) = \sin(x)$ from $x = 0$ to $x = \frac{\pi}{2}$ using four approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is it an underestimate or an overestimate?